From Altruism to Non-Cooperation in Routing Games

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***Abstract*— The paper studies the routing in the network shared by several users. Each user seeks to optimize either its own performance or some combination between its own performance and that of other users, by controlling the routing of its given flow demand. We parameterize the degree of cooperation which allows to cover the fully non-cooperative behavior, the fully cooperative behavior, and even more, the fully altruistic behavior, all these as special cases of the parameter's choice. A large part of the work consists in exploring the impact of the degree of cooperation on the equilibrium. Our first finding is to identify multiple Nash equilibria with cooperative behavior that do not occur in the non-cooperative case under the same conditions (cost, demand and topology). We then identify Braess like paradox (in which adding capacity or adding a link to a network results in worse performance to all users) in presence of user's cooperation. We identify another type of paradox in cooperation scenario: when a given user increases its degree of cooperation while other users keep unchanged their degree of cooperation, this may lead to an improvement in performance of that given user. We then pursue the exploration and carry it on to the setting of Mixed equilibrium (i.e. some users are non atomic-they have infinitesimally small demand, and other have finite fixed demand). We finally obtain some theoretical results that show that for low degree of cooperation the equilibrium is unique, confirming the results of our numerical study.**

I. Introduction

Non-cooperative routing has long been studied both in the framework of road-traffic as well as in the framework of telecommunication networks. Such frameworks allow to model the flow configuration that results in networks in which routing decisions are made in a non-cooperative and distributed manner between the users. In the case of a finite (not very large) number of agents, the resulting flow configuration corresponds to the so called Nash equilibrium [17] defined as a situation in which no agent has an incentive to deviate unilaterally. The Nash equilibrium has been extensively used in telecommunications, see e.g. [2], [6]. The authors in [2] studied a routing games in which each user has a given amount of flow to ship and has several paths through which he may split that flow. Such a routing game may be handled by models similar to [8] in the special case of a topology of parallel links. This type of topology is studied in detail in the first part of [2] as well as in [9]. However, the model of [8] does not extend directly to other topologies. Indeed, in more general topologies, the delay over a *path* depends on how much traffic is sent by other users on any other path that shares common links. Routing games with general topologies have been studied, for example, in the second part of [2], as well as in [9]. A related model was studied thirty years ago by Rosenthal in [10], yet in a discrete setting. It is shown that in such a model there always exists a pure strategy Nash equilibrium. He introduces a kind of discrete potential function for computing the equilibrium. Nevertheless if a player has more than 1 unit to ship such an equilibrium doesn't always exist.

In this work, we embark on experimental investigation of the impact of cooperation in the context of routing games. In particular we consider parallel links and load balancing network topology for investigation, originally presented in [2] and [7] in the context of selfish users. The experimentation is mainly aimed at exploring some strange behaviors which appears in presence of user's partial cooperation (Cooperation in Degree), which is further strengthened with some theoretical results.

Firstly, we identify loss of uniqueness of Nash equilibria. We show by a simple example of parallel links and load balancing network that there may exist several such equilibria. Moreover, even the uniqueness of link utilization at equilibria may fail even in the case of simple topology. A similar example of parallel links, in absence of the cooperation between users there would be a single equilibrium [2]. Beyond Nash equilibrium we investigate further in the setting of Mixed users i.e. where there are two types of users, Group user and Individual users. Group users seek Nash equilibrium while the Individual users seek equilibrium with Wardrop conditions. Strengthening our earlier finding, we observe loss of uniqueness with partial cooperation against the unique solutions shown in [15] for selfish users. However in the latter section (Sec. V), we show theoretically that there exist uniqueness of Nash equilibrium under some conditions in the presence of cooperation between users.

Secondly, we identify paradoxical behavior in presence of such cooperation. One of the observed paradox here is a kind of Braess paradox, a well studied paradox in routing context. Braess paradox has attracted attention of many researchers in context of routing games especially related to upgrading the system, see [4]- [7]. The famous Braess paradox tell us that increasing resources to the system leads to degraded performance in some cases. Such paradox is originally shown to exist in many scenarios, e.g. Braess network in [5], Load balancing network in [7]. Although such paradoxes are found even in

the case of selfish users earlier, their existence even in case of such partial cooperation is highlighted here. We show that as the link capacity increases the overall cost of a user decreases i.e. addition of resources in the system can tentatively lead to degraded performance. Even more, we also identify another kind of paradox related to *degree of Cooperation*: When a user increases its degree of cooperation while other users keep their degree of cooperation unchanged, leads to performance improvement of that user. We also observe similar behavior even when other user also increase their degree of cooperation.

The paper is structured as follows : In section II, we present the system model, define our framework of cooperative user and, formulate the problem. Further in section III we detail the numerical investigation and summarize the findings. Based on one of the findings, we depict more examples identifying Braess paradox in the setting of Nash game in subsection III-C. In section IV, mixed equilibrium is illustrated. In section V, we develop theoretical results to show the conditions where uniqueness can be established in presence of users cooperation. In section VI we summarize the study of impact of cooperation.

II. System model

We consider a network $(\mathcal{V}, \mathcal{L})$, where $\mathcal{V}$ is a finite set of nodes and $\mathcal{L} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of directed links. For simplicity of notation and without loss of generality, we assume that at most one link exists between each pair of nodes (in each direction). For any link $l = (u, v) \in \mathcal{L}$,define $S(l) = u$ and $D(l) = v$. Considering a node $v \in \mathcal{V}$, let $\mathrm{In}(v) = \{l : D(l) = v\}$ denote the set of its in-going links, and $\mathrm{Out}(v) = \{l : S(l) = v\}$ the set of its out-going links.

A set $\mathcal{I} = \{1, 2, ..., I\}$ of users share the network $(\mathcal{V}, \mathcal{L})$, where each source node acts as a user in our frame work. We shall assume that all users ship flow from source node s to a common destination d. Each user i has a throughput demand that is some process with average rate r^i. User i splits its demand r^i among the paths connecting the source to the destination, so as to optimize some individual performance objective. Let f_l^i denote the expected flow that user i sends on link l. The user flow configuration $\mathbf{f}^i = (f_l^i)_{l \in \mathcal{L}}$ is called a routing strategy of user i. The set of strategies of user i that satisfy the user's demand and preserve its flow at all nodes is called the strategy space of user i and is denoted by $\mathbf{F}^i$, that is:

$$\mathbf{F}^i = \{\mathbf{f}^i \in \mathbb{R}^{|\mathcal{L}|}; \sum_{l \in \mathrm{Out}(v)} f_l^i = \sum_{l \in \mathrm{In}(v)} f_l^i + r_v^i, v \in \mathcal{V}\},$$

where $r_s^i = r^i, r_d^i = -r^i$ and $r_v^i = 0$ for $v \neq s, d$. The system flow configuration $\mathbf{f} = (f^1, ..., f^I)$ is called a *routing strategy profile* and takes values in the product strategy space $\mathbf{F} = \otimes_{i \in \mathcal{I}} \mathbf{F}^i$.

The objective of each user i is to find an admissible routing strategy $\mathbf{f}^i \in \mathbf{F}^i$ so as to minimize some performance objective, or cost function, J^i, that depends upon $\mathbf{f}^i$ but also upon the routing strategies of other users. Hence $J^i(\mathbf{f})$ is the cost of user i under routing strategy profile $\mathbf{f}$.

A. Nash equilibrium

Each user in this frame work minimizes his own cost functions which leads to the concept of Nash equilibrium. The minimization problem here depends on the routing decision of other users, i.e., their routing strategy

$$\mathbf{f}^{-i} = (\mathbf{f}^1, ..., \mathbf{f}^{i-1}, \mathbf{f}^{i+1}, ...\mathbf{f}^I),$$

Definition 1 *A vector* $\tilde{\mathbf{f}}^i$, $i = 1, 2, ..., I$ *is called a Nash equilibrium if for each user* i, $\tilde{\mathbf{f}}^i$ *minimizes the cost function given that other users' routing decisions are* $\tilde{\mathbf{f}}^j$, $j \neq i$. *In other words,*

$$J^i(\tilde{\mathbf{f}}^1, \tilde{\mathbf{f}}^2, ..., \tilde{\mathbf{f}}^I) = \min_{\mathbf{f}^i \in \mathbf{F}^i} J^i(\tilde{\mathbf{f}}^1, \tilde{\mathbf{f}}^2, ..., \mathbf{f}^i, ..., \tilde{\mathbf{f}}^I), \quad i = 1, 2, ..., I, \tag{1}$$

where $\mathbf{F}^i$ *is the routing strategy space of user* i.

Nash equilibrium has been discussed in the context of non-cooperative game with selfish users quite often in recent studies.

In this paper we study a new aspect of cooperative routing games where some users cooperate with the system taking into account the performance of other users. We define this *degree of Cooperation* as follows :

Definition 2 *Let* $\overrightarrow{\alpha^i} = (\alpha_1^i, .., \alpha_{|\mathcal{I}|}^i)$ *be the* degree of Cooperation *for user* i. *The new operating cost function* $\hat{J}^i$ *of user* i *with Degree of Cooperation, is a convex combination of the cost of user from set* $\mathcal{I}$,

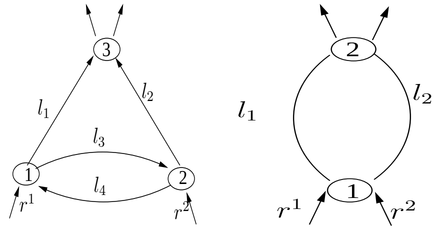
$$\hat{J}^i(\mathbf{f}) = \sum_{k \in \mathcal{I}} \alpha_k^i J^k(\mathbf{f}); \ \sum_k \alpha_k^i = 1, i = 1, ...|\mathcal{I}|$$

where $\hat{J}^i(\boldsymbol{f})$ *is a function of system flow configuration* ***f*** *with cooperation.*

Based on the *degree of Cooperation* vector, we can view the following properties for user i,

- Non cooperative user : if $\alpha_i^i = 0$.
- Altruistic user : User i is fully cooperative with all users and does not care for his benefits, i.e., $\alpha_i^i = 1$.
- Equally cooperative - if $\alpha_j^i = \frac{1}{|\mathcal{P}|}$, user i is equally cooperative with each user j, where $j \in \mathcal{P}, \mathcal{P} \subseteq \mathcal{I}$.

Note that the new operating cost function of a user is the performance measure with *degree of Cooperation*, where it takes into account the cost of other users. Although a user cooperating with the system, it attempts to minimize its own operating cost function in the game setting. Hence such frame work can be classified under non-cooperative games and the thus we can benefit to apply the properties of non-cooperative games in such scenario.



(a) Load balancing (b) Parallel links

Fig. 1. Network Topology

III. Numerical Investigation of the role of cooperation

In this section we detail some numerical examples to study the routing game in the presence of cooperation between some users. In these examples, we use two types of cost functions : linear function which is often used in the transportation network and M/M/1 function which is used in the queuing networks. We consider two network topologies : parallel links [2] and load balancing networks [6] which are defined below

Load Balancing Network: A simple load balancing topology of network $\mathcal{G}$ consists of 3 nodes is depicted in Fig. 1(a). This topology has been widely studied in context of queuing networks. The nodes are numbered $1, 2, 3$ and communication links among them are numbered as l_1, l_2, l_3, l_4. Node $1, 2$ acts as source node and node 3 acts as destination node. Link l_1, l_2 are directed links for nodes $1, 3$ and nodes $2, 3$ where as, link l_3, l_4 are directed link for nodes $1, 2$ and nodes $2, 1$. Cost function of user i is the sum of cost of each link $J^i = \sum_{l\in\{1,\dots 4\}} f_l^i T_l(f_l)$, where $T_l(f_l)$ is the link cost function. The cost of each user i with cooperation can be defined as below,

$$\hat{J}^i = \sum_{l\in\{1,\dots 4\}} \sum_{k\in\{1,2\}} \alpha_k^i f_l^k T_l(f_l) \qquad (2)$$

Parallel Links Network: A simple parallel links topology of network $\mathcal{G}$ consists of 2 nodes is depicted in Fig. 1(b) which is originally discuses in [2]. The nodes are numbered $1, 2$ and communication links between them are numbered as l_1, l_2. Node 1 acts as source node and node 2 acts as destination node. Cost function of user i is the sum of cost of each link $J^i = \sum_{l\in\{1,2\}} f_l^i T_l(f_l)$, where $T_l(f_l)$ is the link cost function. The cost of each user i with cooperation can be defined as below,

$$\hat{J}^i = \sum_{l\in\{1,2\}} \sum_{k\in\{1,2\}} \alpha_k^i f_l^k T_l(f_l) \qquad (3)$$

For each network topology, we consider both the cost functions for investigation.

Linear Cost Function: Linear link cost function is defined as, $T_l(f_{l_i}) = a_i f_{l_i} + g_i$ for link $i = 1, 2$, where as, $T_l(f_{l_j}) = c f_{l_j} + d$ for link $j = 3, 4$.

M/M/1 Delay Cost Function: The link cost function can be defined as, $T_l(f_{l_i}) = \frac{1}{C_{l_i} - f_{l_i}}$, where C_{l_i} and f_{l_i} denote the total capacity and total flow of the link l_i. Note that this cost represents the average expected delay in a M/M/1 queue with exponentially distributed inter arrival times and service times under various regimes such as the FIFO (First In First Out) regime in which customers are served in the order of arrivals, the PS (Processor sharing) regime and the LIFO (Last In First Out) regime. This same cost describes in fact the expected average delays in other settings as well such as the M/G/1 queue (exponentially distributed inter arrival times and general independent service times) under the PS or the LIFO regime.

A. Numerical Examples

We consider two users share a network. We distinguish two cases. An asymmetric case in which the user 1 is cooperative with $\alpha_1^1 > 0$ and user 2 is noncooperative, i.e., $\alpha_2^2 = 0$. The second case is symmetric case in which both users are cooperative with the same degree of cooperation α. We compute the Nash equilibrium at sufficiently many points of *degree of Cooperation* α in the interval [0,1] and plot the corresponding user cost and user flow. Here user flow signifies the fraction of demand flowing in the corresponding user destination link. Since we consider only two links, the fraction of demand flow in one route complements that of the other route. Hence we plot the fraction of demand corresponding to the user, i.e., $f_{l_1}^1$ for user 1 and $f_{l_2}^2$ for user 2. In sequel we describe five experiments as follows:

Experiment 1) Load balancing network with linear link cost: In Fig. 2(a)-2(b), we plot the cost and the flow obtained at Nash equilibrium versus α in the range [0, 1]. Note that the plot of user 1 and 2 overlap in the figure in symmetrical case. This is due to the same degree of Cooperation.

Experiment 2) Parallel links with linear link cost: In Fig. 3(a),3(b), we plot the cost function and the flow for both users obtained at Nash equilibrium for α in the range of [0, 1].

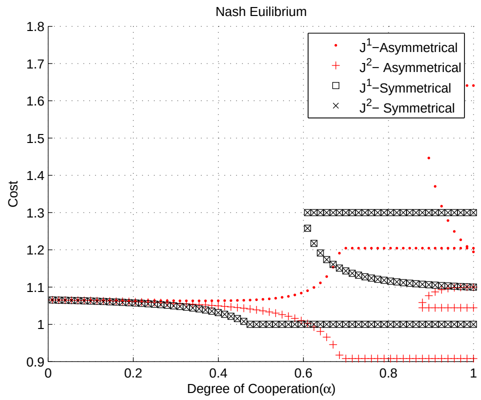
Experiment 3) Load balancing network with M/M/1 link cost: Consider the parameters for the link cost functions as, $a_1 = 4, g_1 = 1, a_2 = 2, g_2 = 2, r^1 = 1.2, r^2 = 1$. In Fig. 4(a),4(b), we plot cost and flow obtained at Nash equilibrium for $0 \le \alpha \le 1$.

Experiment 4) Parallel links with M/M/1 link cost: In Fig. 5(a),5(b), we plot the cost function and the flow for both users obtained at Nash equilibrium versus α.

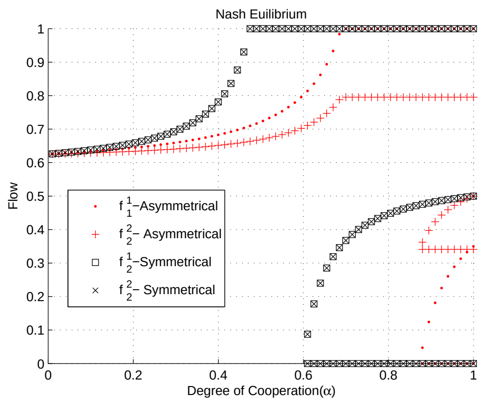
Experiment 5) Load balancing network with linear link cost: We vary the link cost for l_3 and l_4 by varying the parameter c. More precisely, we increase c from 0 to 1000 in the steps of 20 and compute Nash equilibrium at each point. In Fig.6, we plot the cost of each user with the increasing link cost of the link l_3 and l_4. Note that when the link cost is high signifies that link doesn't exit. We analyze the results obtained from the experimentation done above. We will be using α and α^1 alternatively here for simplicity as we have fixed $\alpha^2 = 1$ for asymmetrical case and $\alpha^1 = \alpha^2$ for symmetrical case. The important behavior can be summarized under following two headings.

B. Non uniqueness of Nash equilibrium

In Fig. 2, we observe that there exist **multiple Nash equilibria** for both symmetrical case and asymmetrical case.

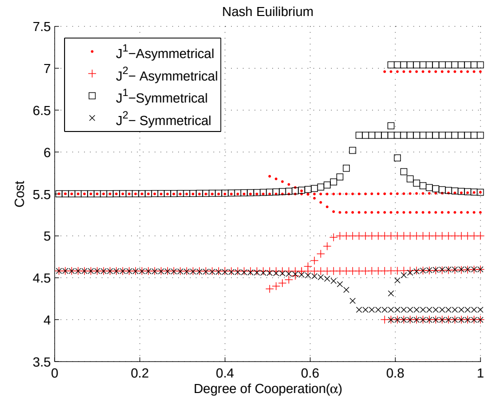


(a) Cost at NEP

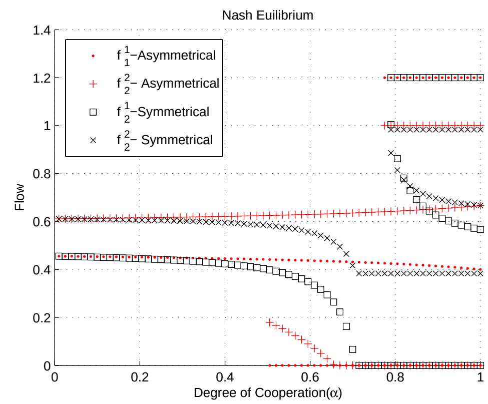


(b) Flow values at NEP

Fig. 2. Topology : Load balancing, Cost function : Linear, Parameters : $a = 1, c = 0, d = 0.5$, Cooperation : { Symmetrical: $\alpha^1 = \alpha^2$, Asymmetrical: $0 \leq \alpha^1 \leq 1, \alpha^2 = 0$}.



(a) Cost function at Nash equilibrium



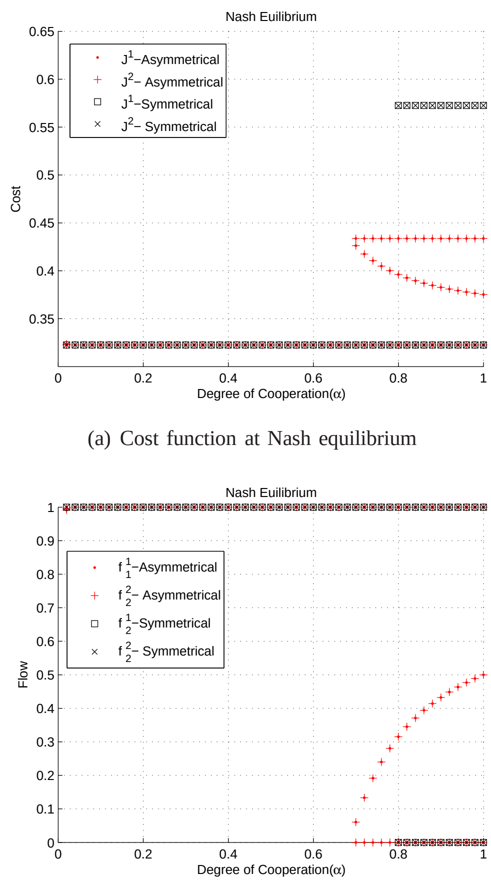
(b) Flow values at Nash equilibrium

Fig. 3. Topology : Parallel links, Cost function : Linear, Parameters : $a = 1, c = 0, d = 0.5$, Cooperation: { Symmetrical: $\alpha^1 = \alpha^2$, Asymmetrical: $0 \leq \alpha^1 \leq 1, \alpha^2 = 0$}.

Note that multiple Nash equilibria is constrained to some range of cooperation(α). However there also exist some range of cooperation where unique solution exist. We observe that there exist three Nash equilibrium for some range of cooperation, two Nash equilibrium at one point and, unique Nash equilibrium for some range of cooperation. In Fig. [4-6](a,b), we obtain multiple Nash equilibria as above for some range of cooperation. In Fig. 3-5 although $\alpha^1 = \alpha^2$, due to other parameter being non-symmetrical, we do not observe a symmetrical plot for "J^1, J^2-Symmetrical". Uniqueness of Nash equilibrium is shown in [2], for a similar situation as in Fig. 3 for selfish user, but we observe loss of uniqueness when users have some cooperation.

C. *Braess like paradox*

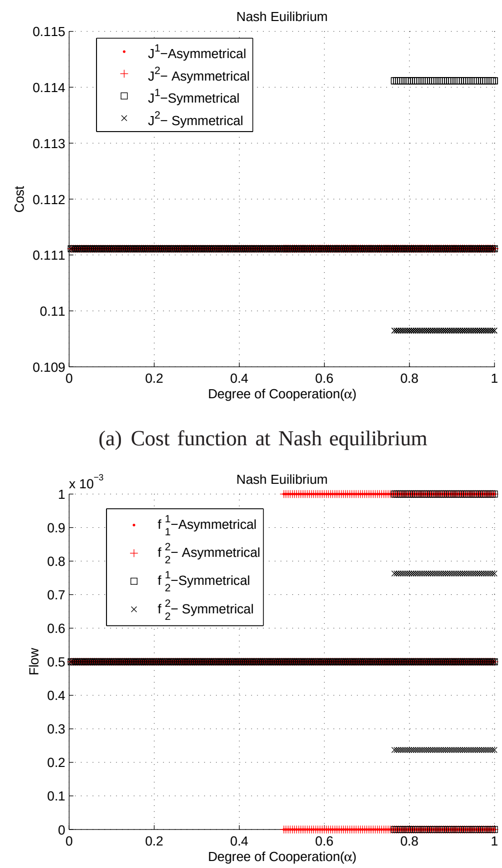
We also observe a Braess kind of paradox which is related to performance when additional resource is added to the system. To understand this, consider the topology of experiment 1, i.e., the load balancing network topology. Consider a configuration where initially link l_3 and l_4 has very high cost (effectively doesn't exist) and latter the link cost is reduced to a low value e.g. $c = 0$ and $d = 0.5$. This can be interpreted as an additional resources added to the system. Observe than for the initial configuration the cost of user 1 is $J^1 = 1$ and cost of user 2 is $J^2 = 1$ in experiment 1. However in the latter configuration which is depicted in Fig. 2(a), we observe the cost of user 1 and 2 is greater that 1 at Nash equilibria. This explains degradation of performance when resources are increased. A very clearer observation can be made in Fig.6 where the link cost for link l_3 and l_4 is parameterized. Due to multiple Nash equilibria we see two curves. The lower curve corresponds to Nash solutions where flow for each user choose direct link to destination while the upper curve correspond to mixed strategy solution where a fraction of flow for each user choose direct link path. Notice that user cost is improving as the link cost is increasing for the upper curve. Such paradox is widely studied as **Braess paradox** in many literature. Above we identified the existence of Braess paradox in load balancing network. Now we identify the Braess paradox in parallel links topology. Consider the parameters as follows, $C_{l_1} = 4.1\, C_{l_2} = 4.1\, r^1 = 2\, r^2 = 1$. Consider the scenario when initially the link l_3, l_4 does not exist, while latter they are added in the system. In other words, the initially the capacity $C_3 = 0, C_4 = 0$, and latter it is $C_3 = 10, C_4 = 10$. Note that when $C_3 = 0, C_4 = 0$,



(a) Cost function at Nash equilibrium

(b) Flow values at Nash equilibrium

Fig. 4. Topology : Load balancing, Cost function : M/M/1 Delay, Parameters : $C_{l_1} = 4.1, C_{l_2} = 4.1, C_{l_3} = 5, C_{l_4} = 5, r^1 = 1, r^2 = 1$, Cooperation: { Symmetrical: $\alpha^1 = \alpha^2$, Asymmetrical: $0 \leq \alpha^1 \leq 1, \alpha^2 = 0$}.



(a) Cost function at Nash equilibrium

(b) Flow values at Nash equilibrium

Fig. 5. Topology : Parallel links, Cost function : M/M/1 Delay, Parameters : $C_{l_1} = 0.001, C_{l_2} = 0.001, r^1 = 1, r^2 = 1$, Cooperation: { Symmetrical: $\alpha^1 = \alpha^2$, Asymmetrical: $0 \leq \alpha^1 \leq 1, \alpha^2 = 0$}.

flow at Nash equilibrium is trivially $f_{l_1} = 1, f_{l_2} = 1$. In the following, we consider two scenarios of degree of cooperation :

Only one user is Cooperative : The degree of Cooperation taken in this case is $\alpha^1 = 0.93, \alpha^2 = 0$. On increasing the capacity C_3, C_4 from $0 \rightarrow 10$, the cost functions at Nash equilibrium are obtained as $J^1 = 0.952 \rightarrow 2.06$, $J^2 = 0.3225 \rightarrow 0.909$ and the flows are $f_{l_1} = 2 \rightarrow 0$, $f_{l_2} = 1 \rightarrow 0.0951$. We also obtain another Nash equilibrium where the cost functions and the flow doesn't change from initial state. Note that increasing the capacity in the network degrades the performance at the first Nash equilibrium.

Both users are Cooperative : We repeat the above experiment with the degree of Cooperation $\alpha^1 = 0.9, \alpha^2 = 0.9$. The cost functions at Nash equilibrium are obtained as $J^1 = 0.952 \rightarrow 1.247$, $J^2 = 0.3225 \rightarrow 0.430$. We again obtain another Nash equilibrium where the cost functions and the flow doesn't change from initial state. Note that again increasing the capacity in the network degrades the performance at the first Nash equilibrium.

D. Paradox in cooperation

In Fig. 2(a), we observe that J^1 has higher cost than J^2. This is intuitive because user 2 is selfish user while user 1 has a varying degree of Cooperation. In particular remark that $\alpha^1 \downarrow 0$, $J^1 \uparrow J^2$. But this is not true for the whole range of Cooperation. Observe in Fig. (3.a) a non intuitive behavior for some small range of α^1 (approximately $\alpha^1 \in (0.87, 1)$. Notice that when the degree of cooperation α^1 increases (i.e. increase in its altruism) while other user be pure selfish $(\alpha^2 = 0)$, leads to improved cost of user 1. This is a paradoxical behavior, we call it **paradox in cooperation**. This paradox also exist in case of symmetrical cooperation (see J^1-*Symmetrical*, J^2-*Symmetrical*) in the range of α approximately $(0, 0.4)$. Notice that such paradox is still observed in Fig. 3-4. Remark that such paradox exist only when there are multiple equilibria.

IV. Mixed Equilibrium

The concept of mixed-equilibrium (M.E.) has been introduced by Harker [14] (and further applied in [16] to a dynamic equilibrium and in [15] to a specific load balancing problem). Harker has established the existence of the M.E., characterized it through variational inequalities, and gave conditions for its

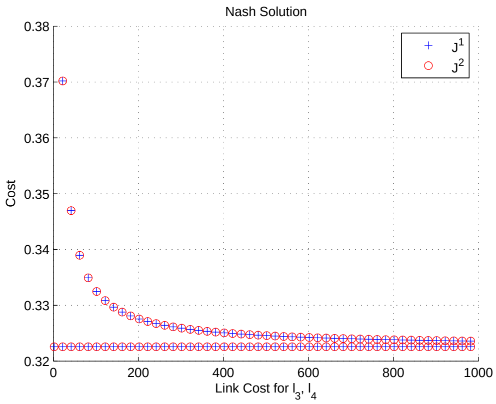


Fig. 6. Topology : Load balancing, Cost function : Linear, Parameters : $a_1 = 4.1, a_2 = 4.1, d = 0.5$, Cooperation:$\alpha^1 = \alpha^2 = 0.93$.

uniqueness. We discuss here the behavior of mixed equilibrium in presence of partial cooperation. Consider the network $(\mathcal{V}, \mathcal{L})$ shared by two types of users: (i) *group users* (denoted by $\mathcal{N}$) : these users have to route a large amount of jobs; (ii) *individual users*; these users have a single job to route through the network form a given source to a given destination. There are infinitely many individual users. For simplicity, we assume that all individual users have a common source s and common destination d. Let $\mathcal{P}$ be the set of possible paths which go from s to d.

Cost function

- $J^i : \mathbf{F} \to [0, \infty)$ is the cost function for each user $i \in \mathcal{N}$
- $\mathcal{F}_p : \mathbf{F} \to [0, \infty)$, is the cost function of path p for each individual user.

The aim of each user is to minimize its cost, i.e., for $i \in \mathcal{N}$, $\min_{f^i} J^i(\mathbf{f})$ and for individual user, $\min_{p\in\mathcal{P}} \mathcal{F}^i_p(\mathbf{f})$. Let fp be the amount of individual users that choose path p.

Definition 3 *$\boldsymbol{f} \in \boldsymbol{F}$ is a Mixed Equilibrium (M.E.) if*

$$\forall i \in \mathcal{N}, \forall g^i s.t. (\boldsymbol{f}^{-i}, g^i) \in \boldsymbol{F}, \hat{J}^i(\boldsymbol{f}) \le \hat{J}^i(\boldsymbol{f}^{-i}, g^i)$$
$$\forall p \in \mathcal{P}, F_{(p)}(\boldsymbol{f}) - A \ge 0;\ (F_{(p)}(\boldsymbol{f}) - A) f^i_{(p)} = 0$$

where $A = \min_{p\in\mathcal{P}} \mathcal{F}_p(\mathbf{f})$

A. Mixed equilibrium in parallel links

In the following proposition, we provide some closed form of Mixed equilibrium in parallel links.

Proposition 1 *Consider parallel links network topology (Fig. 1(b)) and M/M/1 delay link cost function. Consider that a Group type user and Individual type users are operating in this network. The mixed equilibrium strategy $(f^{1^*}_{l_1}, f^{2^*}_{l_2})$ can be given exactly as follows,*

1) *When Both link is used at Wardrop equilibrium:*
$$\begin{cases} (M_1, N_1) & \text{if } a_1 < M_1 < b_1; \\ otherwise & \\ (0, -cc) & \text{if } r_1 < \min\Big(r_2 + C_2 - C_1, \frac{\alpha(C_2-C_1)+2\alpha r_2}{2\alpha-1}\Big), \\ (r_1, r_1 - cc) & \text{if } r_1 < \min\Big(\frac{\alpha(C_2-C_1)}{1-2\alpha}, r_2 - (C_2 - C_1)\Big), \end{cases}$$
where
$$M_1 = \tfrac{-\alpha(C_2-C_1)+r_1(2\alpha-1)}{2(2\alpha-1)},\ N_1 = \tfrac{(C_1-C_2)(1-\alpha)+(2\alpha-1)r_2}{2(2\alpha-1)},$$
$$a_1 = \max(-\tfrac{C_2-C_1}{2} - \tfrac{r_2-r_1}{2}, 0),\ b_1 = \min(-\tfrac{C_2-C_1}{2} + \tfrac{r_1+r_2}{2}, r_1),$$
$$cc = -\tfrac{C_2-C_1}{2} - \tfrac{r_2-r_1}{2},\ dd = -\tfrac{C_2-C_1}{2} + \tfrac{r_2+r_1}{2},$$

2) *When only one link (1) is used at Wardrop equilibrium:*
$$\begin{cases} (M_2, 0) & \text{if } c_1 < M_2 < r_1; \\ otherwise & \\ (c_1, 0) & \text{if } h(r_1) > 0, \\ (r_1, 0) & \text{if } h(r_1) < 0, \end{cases}$$
where $c_1 = \max(-\frac{C_2-C_1}{2} - \frac{r_2-r_1}{2}, 0)$ *and* M_2 *is the unique (if there exists) root of the quadratic equation*
$$h(x) = ax^2 + bx + c = 0$$
in $[c_1, r_1]$. *The coefficients of the quadratic equation are* $a=((C_1-C_2+r_2)(1-\alpha)-\alpha r_2)$; $b=(C_1(1-\alpha)(2(C_2-r_2-r_1)+2(C_2-r_2))+2\alpha r_2 C_1)$; $c=C_1(1-\alpha)[(C_2-r_1-r_2)^2-C_1(C_2-r_2)]-\alpha r_2 C_1^2$.

3) *When only one link (2) is used by Wardrop user:*
$$\begin{cases} (M_3, r_2) & \text{if } 0 < M_3 < d_1; \\ otherwise & \\ (0, r_2) & \text{if } h(0) > 0, \\ (d_1, r_2) & \text{if } h(0) < 0, \end{cases}$$
where $d_1 = \min(-\frac{C_2-C_1}{2} + \frac{r_2+r_1}{2}, r_1)$ *and* M_3 *is the unique root(if there exist) of the quadratic equation*
$$g(x) = ax^2 + bx + c = 0$$
in $[0, d_1]$. *The coefficients of the quadratic equation are* $a=((C_1-C_2+r_2)(1-\alpha)-\alpha r_2)$; $b=(C_1(1-\alpha)$ $(2(C_2-r_2-r_1)+2(C_2-r_2))+2\alpha r_2 C_1)$; $c=C_1(1-\alpha)$ $[(C_2-r_1-r_2)^2-C_1(C_2-r_2)]-\alpha r_2 C_1^2$.

Proof: We first state the general condition for the mixed equilibrium to exist. Based on link uses, there are 3 scenarios when Wardrop conditions can be met for equilibrium to exist. We individually state each of them and then we establish the conditions for equilibria.

For link cost to be finite the link flow must satisfy the flow constraint $f_{l_1} < C_1$, $f_{l_2} < C_2$. From this we obtain the general condition $r_1 + r_2 < C_1 + C_2$. Equilibria can be attained in the following conditions:

1) *When both link is used by Wardrop users:* Wardrop users utilize both the links, i.e., $f^2_{l_1} > 0$, $f^2_{l_2} > 0$, implies cost of both links are same, i.e., $T_{l_1}(f_{l_1}) = T_{l_2}(f_{l_2})$ (we use $T_{l_1}(f_{l_1})$ instead of $\mathcal{F}_{l_1}(f_{l_1})$ from def. 3). From $T_{l_1}(f_{l_1}) = T_{l_2}(f_{l_2}) \Rightarrow f^2_{l_2} = -cc + f^1_{l_1}$, $0 < f^1_{l_1} < r_1$, and $0 < f^2_{l_2} < r_2$ imply that $a_1 \le f^{1^*}_{l_1} \le b_1$, where $a_1 = \max(cc, 0)$, $b_1 = \min(dd, r_1)$, $cc = -\frac{C_2-C_1}{2} - \frac{r_2-r_1}{2}$ and $dd = -\frac{C_2-C_1}{2} + \frac{r_2+r_1}{2}$. Thus the necessary conditions for equilibrium to exist reduces to $r_1 + r_2 > |C_1 - C_2|$ by noting $cc < r_1$ and $dd > 0$. Thus the equilibrium strategy $(f^{1^*}_{l_1}, f^{2^*}_{l_2})$ is given by

$$\begin{cases} (M_1, N_1) & \text{if } a_1 < M_1 < b_1; otherwise, \\ (0, -cc) & \text{if } r_1 < \min\left(r_2+C_2-C_1, \frac{\alpha(C_2-C_1)+2\alpha r_2}{2\alpha-1}\right), \\ (r_1, r_1-cc) & \text{if } r_1 < \min\left(\frac{\alpha(C_2-C_1)}{1-2\alpha}, r_2-(C_2-C_1)\right), \end{cases}$$

where

$$M_1 = \frac{-\alpha(C_2-C_1)+r_1(2\alpha-1)}{2(2\alpha-1)},\ N_1 = \frac{(C_1-C_2)(1-\alpha)+(2\alpha-1)r_2}{2(2\alpha-1)}.$$

Note that $J^1(f^1_{l_1}, f^2_{l_2})$ is strict convex in the range $0 < f^1_{l_1} < r_1, 0 < f^{2^*}_{l_2} < r_2$(by definition of M/M/1 cost function). It can be directly inferred that if the equilibrium point (M_1, N_1) satisfies the condition $a_1 < M_1 < b_1$, (it is an interior point) there exist atmost one equilibrium.

Otherwise when there is no interior equilibrium point, there may exist equilibrium at $fl^1_1 = 0$ or $fl^1_1 = r_1$, i.e at point $(0, -cc)$ or at point $(r_1, r_1 - cc)$ (since $Tl_1(f_{l_1}) = Tl_2(f_{l_2})$ implies $f^2_{l_2} = -cc + fl^1_1$). The point $(0, -cc)$ can be an equilibrium point only when $a_1 = \max(0, cc) = 0$ and $J'^1(0, cc) > 0$. This directly implies $r_1 < r_2 + (C_2 - C_1)$, and $r_1 < \frac{\alpha(C_2-C_1)+2\alpha r_2}{2\alpha-1}$ respectively. Combining these, we get $r_1 < \min\left\{r_2 + (C_2 - C_1), \frac{\alpha(C_2-C_1)+2\alpha r_2}{2\alpha-1}\right\}$. Following the similar steps we can directly obtain that point $(r_1, r_1 - cc)$ can be an equilibrium point when $r_1 < \min\left\{\frac{\alpha(C_2-C_1)}{1-2\alpha}, r_2 - (C_2 - C_1)\right\}$.

2) *When only one link (link 1) is used by Wardrop user:* In this case, Wardrop users utilize only link 1, i.e., $f^2_{l_2} = 0$. This directly implies $T_{l_1}(f_{l_1}) \leq T_{l_2}(f_{l_2}) \Rightarrow f^1_{l_1} \leq cc$ (from wardrop condition). Combining the above with positive flow condition $0 \leq f^{1^*}_{l_1} \leq r_1$, we obtain $0 \leq f^{1^*}_{l_1} \leq c_1$, where $c_1 = \min\{cc, r_1\}$. Since c_1 must be greater than 0, the necessary condition for equilibrium to exist reduces to $r_1 - r_2 \geq C_1 - C_2$. Further the equilibrium strategy $(f^{1^*}_{l_1}, f^{2^*}_{l_2})$ is given by
$$\begin{cases} (M_2, 0) & \text{if } 0 < M_2 < c_1; \\ otherwise, & \\ (0,0) & \text{if } h(0) > 0, \\ (c_1, 0) & \text{if } h(0) < 0, \end{cases}$$
where M_2 is the unique root of quadratic equation $h(x) = ax^2 + bx + c$. Let $x_1 = \frac{-b+\sqrt{D}}{2a}, x_2 = \frac{-b-\sqrt{D}}{2a}$ are the roots of the Quadratic equation $h(x) = 0$, where $a = (C_1 - C_2 - r_2)(1-\alpha) + \alpha r_2;\ b = 2(1-\alpha)[(C_1 - r_2)(2(C_2 - r_2) + r_1)] + 2\alpha r_2(C_2 - r_1);\ c = (1-\alpha)(C_1 - r_2)[(C_2 - r_1)^2 - (C_2 - r_1)(C_1 - r_2) - r_1(C_1 - r_2)] + \alpha r_2(C_2 - r_1)^2;\ D = b^2 - 4ac$.
The quadratic equation $h(x) = 0$ will have unique solution in the range $0 < f^1_{l_1} < r_1$ because $J'^1(f^1_1, 0)$ is strict convex in the range $0 < f^1_{l_1} < r_1$ (by definition of M/M/1 cost function). Hence there can be atmost one equilibrium point satisfying $0 < M_2 < c_1$(i.e single interior point).
Otherwise when there is no interior equilibrium point, there may exist equilibrium at $fl^1_1 = 0$ or $fl^1_1 = r_1$, i.e., at point $(0,0)$ or at point $(r_1, 0)$. The point $(0,0)$ can be an equilibrium point only when $J'^1(0,0) > 0$, i.e., $h(0) > 0$. Similarly point $(c_1, 0)$ can be equilibrium point only when $J'^1(0,0) < 0$, i.e., $h(0) < 0$.

3) *When only one link (2) is used by Wardrop user:* In this case Wardrop users utilize only link 2, i.e., $f^2_{l_2} = r_2$. Following the similar steps as before, we obtain $d_1 \leq f^{1^*}_{l_1} \leq r_1$, where $d_1 = \max\{dd, 0\}$. Since d_1 must be less than r_1, the necessary condition for equilibrium to exist reduces to $r_1 - r_2 \leq C_2 - C_1$.
Further the equilibrium strategy $(f^{1^*}_{l_1}, f^{2^*}_{l_2})$ is given by
$$\begin{cases} (M_3, r_2) & \text{if } d_1 < M_3 < r_1; \\ otherwise & \\ (0, r_2) & \text{if } h(r_2) > 0, \\ (d_1, r_2) & \text{if } h(r_2) < 0, \end{cases}$$
where M_3 is the unique root(if there exist) of the quadratic equation $g(x) = ax^2 + bx + c$ in $d_1 < f^1_{l_1} < r_1$. Let $x_1 = \frac{-b+\sqrt{D}}{2a}, x_2 = \frac{-b-\sqrt{D}}{2a}$ are the roots of the Quadratic equation $g(x) = 0$, where $a = ((C_1 - C_2 + r_2)(1-\alpha) - \alpha r_2);\ b = (1-\alpha)[4C_1(C_2 - r_1 - r_2) + 2r_1C_1] - 2\alpha r_2 C_1);\ c = (1-\alpha)[(C_2 - r_1 - r_2 + C_1)C_1(C_2 - r_2 - r_1) - r_1 C_1^2] + \alpha r_2 C_1^2;\ D = b^2 - 4ac$.
The quadratic equation $g(x) = 0$ will have unique solution in the range $0 < f^1_{l_1} < r_1$ because $J'^1(f^1_1, r_2)$ is strict convex in the range $0 < f^1_{l_1} < r_1$ (by definition of M/M/1 cost function). Hence there can be atmost one equilibrium point satisfying $d_1 < M_3 < r_1$(i.e single interior point).
Otherwise when there is no interior equilibrium point, there may exist equilibrium at $fl^1_1 = 0$ or $fl^1_1 = r_1$, i.e., at point $(0, r_2)$ or at point (r_1, r_2). The point $(0, r_2)$ can be an equilibrium point only when $J'^1(0, r_2) > 0$, i.e., $g(r_2) > 0$. Similarly point (r_1, r_2) can be equilibrium point only when $J'^1(0, r_2) < 0$, i.e., $g(r_2) < 0$.

Corollary 1 *Consider the symmetric parallel links, i.e., $(C_1 = C_2 = C, r_1 = r_2 = r)$ network with M/M/1 delay link cost function. In a mixed user setting the mixed equilibrium strategy $((f^{1^*}_{l_1}, f^{2^*}_{l_2}))$ can be given by*
$$\begin{cases} (\frac{r}{2}, \frac{r}{2}) & \textit{when } r_1 > f^1_{l_1} > 0, r_2 > f^2_{l_2} > 0 \\ (0,0) & \textit{when } 0 \leq f^1_{l_1} \leq r_1, f^2_{l_2} = 0,\ \textit{if } \alpha \geq 0.5 \\ (r, r) & \textit{when } 0 \leq f^1_{l_1} \leq r_1, f^2_{l_1} = 0,\ \textit{if } \alpha \geq 0.5 \end{cases}$$

Proof:

Consider the symmetric case when $C_1 = C_2 = C$, $r_1 = r_2 = r$. The general condition thus reduces to $r < C$ from prop. 1. Equilibrium can be attained under the following scenario based on link uses.

1) *When both link is used by Wardrop users:* Wardrop users utilizes both the links, i.e., $f^2_{l_1} > 0, f^2_{l_2} > 0$, implies cost function of both the links are same, i.e., $T_{l_1}(f_{l_1}) = T_{l_2}(f_{l_2})$. From $T_{l_1}(f_{l_1}) = T_{l_2}(f_{l_2}) \Rightarrow f^2_{l_2} = f^1_{l_1}$, $0 < f^1_{l_1} < r$, and $0 < f^2_{l_2} < r$, implies that necessary condition for equilibrium to exist are always satisfied. Further the equilibrium strategy $(f^{1^*}_{l_1}, f^{2^*}_{l_2})$ is given by $(\frac{r}{2}, \frac{r}{2})$ which

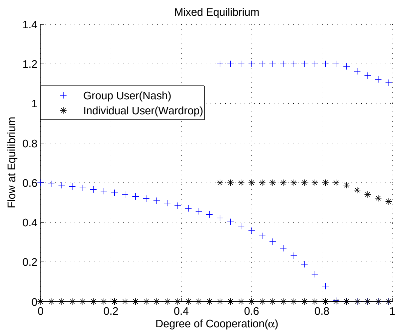


Fig. 7. Topology : Parallel links, Cost function : M/M/1 Delay, Parameters : $C_{l_1} = 4, C_{l_2} = 3, r^1 = 1.2, r^2 = 1$.

can be directly obtained from prop. (1.1).

2) *When only one link (link 1) is used by Wardrop user:* In this case, Wardrop users utilize only link 1, i.e., $f^2_{l_2} = 0$. This directly implies $T_{l_1}(f_{l_1}) \leq T_{l_2}(f_{l_2}) \Rightarrow f^1_{l_1} \leq 0$ (from wardrop condition). Combining the above with positive flow condition $0 \leq f^{1^*}_{l_1} \leq r_1$, we obtain $f^{1^*}_{l_1} = 0$. This suggests that equilibrium point can be given by $(f^{1^*}_{l_1}, f^{2^*}_{l_2}) = (0, 0)$ if there exist.
Note that $(0, 0)$ is the boundary point solution. If $J'^1(0, 0) \geq 0$ (Nash solution of user 1) then the equilibrium point is given by $(0, 0)$. $J^{1'}(f^1_{l_1}, 0)$ can be expressed as $\frac{P(x)}{Q(x)}$, where
$$P(x) = ax^2 + bx + c$$
$a = r(2\alpha-1)$; $b = 2(C-r)(2(C-r)(1-\alpha)+r)$; $c = (2\alpha-1)r(C-r)^2$; $D = 16(C-r)^2(1-\alpha)C[(C-r)(1-\alpha)+\alpha r]$ and $Q(x) > 0$ for all x, thence $J'^1(0, 0) \geq 0 \Rightarrow c \geq 0 \Leftrightarrow \alpha \geq 0.5$.
3) *When only one link (2) is used by Wardrop user:* In this case, Wardrop users utilize only link 2, i.e., $f^2_{l_2} = r$. This directly implies $T_{l_1}(f_{l_1}) \geq T_{l_2}(f_{l_2}) \Rightarrow f^1_{l_1} \geq r$ (from Wardrop condition). Combining the above with positive flow condition $0 \leq f^{1^*}_{l_1} \leq r_1$, we obtain $f^{1^*}_{l_1} = r$. This suggests that equilibrium point can be given by $(f^{1^*}_{l_1}, f^{2^*}_{l_2}) = (r, r)$ if there exist.
Remark that this case is symmetrical to case when only link 1 is used. Hence we can directly infer the condition for equilibrium point to exist. The equilibrium point point (r, r) exist, when $\alpha \geq 0.5$.

In Fig. (7), we depict the mixed equilibrium strategy(flow) for the varying degree of cooperation(α). Observe the loss of uniqueness of mixed equilibrium in presence of partial cooperation. It is known to have unique equilibrium in the network setting with finitely many selfish users [15]. Remark that we have already shown in the previous section that there exist multiple Nash equilibria in presence of partial cooperation. Due to space limitation we illustrate this behavior with only parallel links topology and M/M/1 cost function. However we identify a similar remark from other configuration also.

V. Existence and uniqueness of Equilibrium

Having noted the existence of multiple Nash equilibrium in sec.III-A using various examples, we here establish the conditions under which unique nash equilibrium exist. Uniqueness of Nash equilibrium is shown in [2] in case of non-cooperative games for parallel links topology. Under some condition, uniqueness is shown for general topology also. In this section we follow the similar structure to establish the uniqueness for parallel links topology in case of our setting of user cooperation.

We follow some assumptions on the cost function J^i same as in [2].

Assumption V.1 *:*
G1: $J^i(\mathbf{f}) = \sum_{l \in \mathcal{L}} \hat{J}^i_l(f_l))$. *Each* $\hat{J}^i_l$ *satisfies:*
G2: J^i_l:$[0, \infty) \rightarrow (0, \infty]$ *is continuous function.*
G3: J^i_l*: is convex in* f^j_l *for* $j = 1, ...|\mathcal{I}|$.
G4: *Wherever finite,* J^i_l *is continuously differentiable in* f^i_l, *denote* $K^i_l = \frac{\delta \hat{J}^i_l}{\delta f^i_l}$.

Note the inclusion of $+\infty$ in the range of $\hat{J}^i_l$, which is useful to incorporate implicitly and compactly and additional constraints such as link capacities. Also note that the assumption $G3$ is stronger than in [2].

Function that comply with these general assumptions, we call type G function. For selfish user operating on parallel links NEP is shown to exist in [2] with the function which comply with the type G function.

We shall mainly consider cost functions that comply with the following assumptions:

Assumption V.2 *:*
B1: $J^i(\mathbf{f}) = \sum_{l \in \mathcal{L}} f^i_l T_l(f_l))$
B2: $T_l : [0, \infty) \rightarrow (0, \infty]$.
B3: $T_l(f_l)$ *is positive, strictly increasing and convex.*
B4: $T_l(f_l)$ *is continuously differentiable.*

Functions that comply with these assumptions are referred to as *type*-**B** functions.

Remark 1 *In Assumption* **B1**, $T_l(f_l)$ *is the cost per unit of flow (for example mean delay) on the link* l, *for the total utilization,* $f_l = \sum_{i \in \mathcal{I}} f^i_l$, *of that link. Note that if* $T_l(f_l)$ *is the average delay on link* l, *it depends only on the total flow on that link. The average delay should be interpreted as a general congestion cost per unit of flow, which encapsulates the dependence of the quality of service provided by a finite capacity resource on the total load* f_l *offered to it.*

A special kind of type-B cost function is that which corresponds to an M/M/1 link model. In other words, suppose that
C1: $\hat{J}^i(f^i_l, f_l) = f^i_l \dot{T}_l(f_l)$ is a type-B cost function.
C2: $T_l = \begin{cases} \frac{1}{C_l - f_l} & f_l < C_l \\ \infty & f_l > C_l \end{cases}$.
Where C_l is the capacity of the link l.

Function that comply with these requirements are referred to as type-**C** functions. Such delay functions are broadly used in modeling the behavior of the links in computer communication networks [11], [12].

A. *Parallel links network topology*

In this section we study the special case where the users from set $\mathcal{I}$ shares a set of parallel communication links $\mathcal{L} = \{1, 2....L\}$ interconnecting a common source node to a common destination node. In [2], uniqueness of Nash equilibrium is shown for the selfish users (when user do not cooperate in managing the communication link) in parallel links, where the cost functions ($J^i(\mathbf{f})$) of users are assumed to hold assumption V.2. However this is not true when the users have cooperation in degree as defined in sec.(2). We observe that assumption V.2 is not sufficient to guarantee unique Nash equilibrium in our setting. It is a harder problem to characterize system behavior for general degree of cooperation. Hence we consider a special case of cooperation where a user cooperative with similar cooperation with all other users i.e.

$$\hat{J}^i(\mathbf{f}) = (1-\alpha^i)J^i(\mathbf{f}) + \alpha^i \sum_k J^k(\mathbf{f})$$

Consider the cost function of type V.2. The cost function of each user on link l is given by

$$\begin{aligned} \hat{J}^i_l(\mathbf{f}) &= ((1-\alpha^i)f^i_l + \alpha^i f^{-i}_l)T_l(f_l) \\ &= ((1-\alpha^i)f_l + (1-2\alpha^i)f^{-i}_l)T_l(f_l) \end{aligned}$$

Existence problem in the case of Nash equilibrium for the cost function $\hat{J}^i_l(\mathbf{f})$ can be directly studied as in [2].

Note that in case of $\alpha^i < 0.5$ for all $i \in \mathcal{I}$, the uniqueness of Nash equilibrium is guaranteed from E. Orda et al. [2]. Note that when $\alpha^i < 0.5$, the function $K^i_l(f^{-i}_l, f_l)$ is strictly increasing function in f^{-i}_l and f_l.

Uniqueness of Nash equilibrium can be also observed in case of All-positive flow in each link. By All-positive flow we mean that each user have strictly positive flow on each link of the network.

The following result establishes the uniqueness of Nash Equilibrium in case of positive flow.

Theorem V.1 *Consider the cost function of type V.2. Let $\hat{\mathbf{f}}$ and $\mathbf{f}$ be two Nash equilibria such that there exists a set of links $\overline{\mathcal{L}}_1$ such that $\{f^i_l > 0$ and $\hat{f}^i_l > 0, i \in \mathcal{I}\}$ for $l \in \overline{\mathcal{L}}_1$, and $\{f^i_l = \hat{f}^i_l = 0, i \in \mathcal{I}\}$ for $l \notin \overline{\mathcal{L}}_1$. Then $\hat{\mathbf{f}} = \mathbf{f}$.*

Proof: Let $\mathbf{f} \in F$ and $\hat{\mathbf{f}} \in F$ be two NEP's. As observed $\mathbf{f}$ and $\hat{\mathbf{f}}$ satisfy the Kuhn-Tucker condition. We rewrite the Kuhn-Tucker condition in terms of f^{-i}_l, f_l as below,

$$\begin{aligned} K^i_l(f^{-i}_l, f_l) &\geq \lambda^i; K^i_l(f^{-i}_l, f_l) = \lambda^i \text{ if } f^i_l > 0\ \forall i, l \\ K^i_l(\hat{f}^{-i}_l, \hat{f}_l) &\geq \lambda^i; K^i_l(\hat{f}^{-i}_l, \hat{f}_l) = \lambda^i \text{ if } \hat{f}^i_l > 0\ \forall i, l \end{aligned}$$

The above relation and the fact that $K^i_l(.;.)$ is increasing in both of is argument will be used below to establish that $\mathbf{f} = \hat{\mathbf{f}}$ i.e. $f^i_l = \hat{f}^i_l$ for every l, i. The first step is to establish that $f_l = \hat{f}_l$ for each link l. To this end, we prove that for each l and i, the following relation holds:

$$\{\hat{\lambda}^i \leq \lambda^i, \hat{f}_l \geq f_l\} \qquad \text{implies that } \hat{f}^{-i} \leq f^{-i}, \tag{4}$$
$$\{\hat{\lambda}^i \geq \lambda^i, \hat{f}_l \leq f_l\} \qquad \text{implies that } \hat{f}^{-i} \geq f^{-i}. \tag{5}$$

We shall prove (4), since (5) is symmetric. Assume that $\hat{\lambda}^i \leq \lambda$ and $\hat{f}_l \geq f_l$ for some l and i. For $f^i_l > 0$ together with our assumptions imply that:

$$K^i_l(\hat{f}^{-i}_l, f_l) = \hat{\lambda}^i \leq \lambda^i \leq K^i_l(f^{-i}_l, f_l) \leq K^i_l(\hat{f}^{-i}_l, \hat{f}_l), \tag{6}$$

where the last inequality follows from the monotonicity of K^i_l in its second argument. Now , since K^i_l is nondecreasing in its first argument, this implies that $f^{-i}_l \leq f^i_l$, and (4) is established.

Let $\mathcal{L}_1 = \{l : \hat{f}_l > f_l\}$. Also denote $\mathcal{I}_a = \{i : \hat{\lambda}^i > \lambda^i\}$, $\mathcal{L}_2 = \mathcal{L} - \mathcal{L}_1 = \{l : \hat{f}^l \leq f_l\}$. Assume that $\mathcal{L}_1$ is non empty. Recalling that $\sum_l \hat{f}^{-i}_l = \sum_l f^{-i}_l = r^{-i}$, it follows from (5) that for every i in $\mathcal{I}_a$,

$$\sum_{l \in \mathcal{L}_1} \hat{f}^{-i} = r^{-i} - \sum_{l \in \mathcal{L}_2} \hat{f}^{-i} \leq r^{-i} - \sum_{l \in \mathcal{L}_2} f^{-i} = \sum_{l \in \mathcal{L}_1} f^{-i}, \ \ i \in \mathcal{I}_a.$$

From (4), we know that , $\hat{f}^{-i}_l \leq f^{-i}_l$ for $l \in \mathcal{L}_1$ and $i \notin \mathcal{I}_a$, it follows that :

$$\sum_{l \in \mathcal{L}_1} \hat{f}_l = \sum_{l \in \mathcal{L}_1} \frac{\sum_{i \in \mathcal{I}} \hat{f}^{-i}_l}{\mathcal{I} - 1} \leq \sum_{l \in \mathcal{L}_1} \frac{\sum_{i \in \mathcal{I}} f^{-i}_l}{\mathcal{I} - 1} = \sum_{l \in \mathcal{L}_1} f_l$$

This inequality obviously contradicts our definition of $\mathcal{L}_1$. Which implies that $\mathcal{L}_1$ is an empty set. By symmetry, it may also be concluded that the set $\{l : \hat{f}_l < f_l\}$ is also empty. Thus, it has been established that:

$$\hat{f}_l = f_l \ \text{ for every } \ l \in \mathcal{L}. \tag{7}$$

We now show that $\hat{\lambda}^i = \lambda^i$ for each user i. To this end, note that (4) may be strengthen as follows:

$$\begin{aligned} &\{\hat{\lambda}^i < \lambda^i, \lambda f_l = f_l\} \text{ implies that either} \\ &\qquad \hat{f}^{-i}_l < f^{-i}_l \text{ or } \hat{f}^{-i}_l = f^{-i}_l = 0. \end{aligned} \tag{8}$$

Indeed if $f^{-i}_l = 0$, then the implication is trivial. Otherwise, if $f^{-i}_l > 0$, it follows similar to (6) that $K^i_l(\hat{f}^{-i}_l, \hat{f}_l)$ that $\hat{f}^{-i}_l < f^{-i}_l$ as required. Assume now that $\hat{\lambda}^i < \lambda^i$ for some $i \in \mathcal{I}$. Since $\sum_{l \in \mathcal{L}} \hat{f}^{-i}_l = r^{-i} > 0$, then $f^{-i}_l > 0$ for at least one link l and from (8) implies that, $\sum_{l \in \mathcal{L}} f^i_l > \sum_{l \in \mathcal{L}} \hat{f}^i_l = r^i$, which contradicts the demand constraint for user i. We, therefore, conclude that $\hat{\lambda}^i < \lambda^i$ does not hold for any user i. A symmetric argument may be used to show that $\hat{\lambda}^i = \lambda^i$ for every user $i \in \mathcal{I}$. Combined with (7), this implies by (4) and (5) that $\hat{f}^{-i}_l = f^{-i}_l$ for every l, i. Again since $f^i_l = f_l - f^{-i}_l$, uniqueness of f^i_l is proved.

B. *Uniqueness of NEP in general topology*

It is a hard to characterize system behavior for general network with user's partial cooperation. For selfish users, it is shown that there exist uniqueness for Nash equilibrium point(NEP) under Diagonal Strict Convexity in [2].

We consider a special case of cooperation where a user cooperates equally with all other users i.e.

$$\hat{J}^i(\mathbf{f}) = (1-\alpha^i)J^i(\mathbf{f}) + \alpha^i \sum_k J^k(\mathbf{f})$$

Consider the cost function of type V.2. The cost function of each user on link l can be thus given by

$$\hat{J}^i_l(\mathbf{f}) \quad = \quad ((1-\alpha^i)f_l + (1-2\alpha^i)f_l^{-i})T_l(f_l) \qquad (9)$$

Theorem V.2 *Consider the cost function of type V.2. Let* $\hat{\mathbf{f}}$ *and* $\mathbf{f}$ *be two Nash equilibria such that there exists a set of links* $\overline{\mathcal{L}}_1$ *such that* $\{f_l^i > 0$ *and* $\hat{f}_l^i, i \in \mathcal{I}\}$ *for* $l \in \overline{\mathcal{L}}_1$*, and* $\{f_l^i = \hat{f}_l^i = 0, i \in \mathcal{I}\}$ *for* $l \notin \overline{\mathcal{L}}_1$*. Then* $\hat{\mathbf{f}} = \mathbf{f}$.

Under all positive flows assumption, the Kuhn-Tuker conditions for all $l = (u,v) \in \mathcal{L}_\infty$ becomes

$$((1-\alpha^i)f_l^i + \alpha^i f_l^{-i})T_l'(f_l) + (1-\alpha^i)T_l(f_l) = \lambda_u^i - \lambda_v^i$$
$$((1-\alpha^i)\hat{f}_l^i + \alpha^i \hat{f}_l^{-i})T_l'(\hat{f}_l) + (1-\alpha^i)T_l(\hat{f}_l) = \hat{\lambda}_u^i - \lambda_v^i$$

Summing each of these equations over i, we obtain

$$H_{uv}(f_l) := (\alpha I + 1 - 2\alpha)T_l'(f_l) + I(1-\alpha)T_l(f_l) = \lambda_u - \lambda_v$$
$$H_{uv}(\hat{f}_l) := (\alpha I + 1 - 2\alpha)\hat{f}_l T_l'(\hat{f}_l) + (1-\alpha)I T_l(\hat{f}_l) = \hat{\lambda}_u - \lambda_v$$

Since the function H is strictly increasing, we follow the same proof of Theorem 3.3 in [2] to conclude that $\hat{\mathbf{f}} = \mathbf{f}$.

VI. Concluding Remarks

This paper is aimed at exploring user performance in routing games where a finite number of users take into account not only their performance but also other's user's performance. We have parameterized the *degree of Cooperation* to capture the user behavior from altruistic to ego-centric regime. We notice some strange behaviors. Firstly we show the existence of multiple Nash equilibria by a simple example of parallel links and load balancing networks in contrast to the unique Nash equilibrium in case of selfish users. Moreover, we then explored the mixed user scenario, which is composed of a finite number of Group type user seeking Nash equilibrium and infinitely many Individual type users satisfying Wardrop condition. We illustrate loss of uniqueness of equilibrium even in mixed users scenario in presence of partial cooperation by an example for parallel links network. However it is known to have unique equilibrium in presence of only finitely many selfish users in similar settings.

Secondly we identify two kinds of paradoxical behavior. We identify situation where well known Braess paradox occurs in our setting of cooperation. We show using an example of parallel links network with M/M/1 link cost that addition of system resources indeed degrades the performance of all users in presence of some cooperation, while it is well known that this is not true for this setting with only selfish users.

We also identify another type of paradox, paradox in cooperation: i.e. when a given user increases its degree of cooperation while other users keep unchanged their degree of cooperation, this may lead to an improvement in performance of that given user. In extreme sense a user can benefit itself by adopting altruistic nature instead of selfishness.

References

[1] Amar Prakash Azad, Eitan Altamn, and Rachid Elazouzi, "From Altruism to Non-Cooperation in Routing Games", available on Arxiv(arXiv:0808.4079).

[2] Ariel Orda, Raphael Rom, and Nahum Shimkin, " Competitive Routing in Multiuser Communication Networks",*IEEE/ ACM Transactions on Networking*, Vol.1 No. 5, October 1993

[3] J.B.Rosen, " Existence and uniqueness of Equilibrium points for concave N-Person Games". *Econometrica* , Vol. 33 , No. 3, Pp. 153-163, July 1965.

[4] N. G. Bean , F. P. Kelly and P. G.Taylor, "Braess' paradox in a loss network". *J. Appl. Prob.* 34, pp. 155–159, 1997.

[5] H. Kameda, "How harmful the paradox can be in Braess/Cohen-Kelly-Jeffries networks", *Proc. IEEE Infocom 2002*, New York , pp. 437-445, June 2002.

[6] H. Kameda , E. Altman and T. Kozawa," A case where a paradox like Braess's occurs in the Nash equilibrium but does not occur in the Wardrop equilibrium - a situation of load balancing in distributed computer systems" , *Proceedings of IEEE CDC'99* , Phoenix, Arizona, USA, Dec. 1999

[7] H. Kameda , E. Altman, T. Kozawa, Y. Hosokawa , "Braess-like Paradoxes in Distributed Computer Systems" , *IEEE Transaction on Automatic control*, Vol 45, No 9, pp. 1687-1691, 2000.

[8] K. Park, M. Sitharam, and S. Chen. "Quality of service provision in noncooperative networks with diverse user requirements", *Decis. Support Syst.*, vol. 28, no.1-2, pp. 101-122, 2000.

[9] E. Altman and H. Kameda. "Equilibria for multiclass routing problems in multi-agent networks", *40th IEEE Conference on Decision and Control*, Orlando, Dec. 2000.

[10] R. W. Rosenthal. A class of games possessing pure strategy nash equilibria. *Int. J. Game Theory*, 2:65–67, 1973.

[11] D. Bertsekas and R. Gallager, " Data Networks". Eaglewood Cliffs. NJ:Prentice Hall, 1987

[12] L. Kleinrock," Queueing Systems", New York: Wiley, 1975.

[13] T. Jimenez, E. Altman, T. Basar and N. Shimkin, "Competitive routing in networks with polynomial

[14] P. Harker, "Multiple equilibrium behaviors on networks", *Transportation Research B*, 22, pp. 39-46, 1988.

[15] T. Boulogne , E. Altman, O. Pourtallier and H. Kameda , "Mixed Equilibrium for Multiclass Routing Games", *IEEE Trans. on Automatic Control, special issue on control issues in telecommunication networks*, Vol 47 Number 6, pp. 903-916, June 2002.

[16] B. W. Wie, "A differential game approach to the dynamic mixed behavior traffic network equilibrium problem", *European Journal of Operational Research*, 83:117136, 1995.

[17] T. Basar and G. Olsder, *Dynamic Noncooperative Game Theory*, Academic Press, London, 1982.